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Sakharov's induced gravity and the Poincaré gauge theory

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Abstract

We explore Sakharov's seminal idea that gravitational dynamics is induced by the quantum corrections from the matter sector. This was the starting point of the view that gravity has an emergent origin, which soon gained impetus due to the advent of black hole thermodynamics. In the generalized framework of Riemann–Cartan spacetime with both curvature and torsion, the induced gravitational action is obtained for free nonminimally coupled scalar and Dirac fields. For a realistic matter content, the induced Newton constant is obtained to be of the magnitude of the ultraviolet cutoff, which implies that the cutoff is of the order of the Planck mass. Finally, we conjecture that the action for any gauge theory of gravity at low energies can be induced by Sakharov's mechanism. This is explicitly shown by obtaining the Poincaré gauge theory of gravity.

Contribution to the Memorial Volume for Jacob Bekenstein

1 Introduction

The standard approach for dealing with ultraviolet-divergent quantum corrections in quantum field theory is renormalization. Its techniques are well-developed mathematically, and the running of coupling constants is verified experimentally for the Standard Model. In curved spacetime, however, we encounter some problems. Particularly, the gravitational couplings are radiatively unstable in the sense that the cosmological constant and the Newton constant are extremely sensitive to any change of the parameters of the matter sector or a change of the Wilsonian cutoff scale of the matter action.

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This problem could likely be solved by imposing constraints that keep certain matter parameters and quantities of the effective action of matter invariant under phase transitions and spontaneous symmetry breaking. The implementation of such constraints requires new physics beyond the Standard Model. Here we explore an alternative and historically significant approach to quantum corrections, which uses a cutoff scale that is related to the observed gravitational couplings and offers an illuminating view on the origin of gravity.

In Sakharov’s approach to induced gravity [1], classical gravity is considered to be induced by quantum corrections from the matter sector. At first matter fields are regarded to live on a spacetime that is curved but with a nondetermined geometry. The quantization of matter fields produces correction terms in the action, which involve the curvature of spacetime. In particular, one-loop corrections include the Einstein–Hilbert action of General Relativity (GR). Finally, elevating the nondetermined metric of spacetime to a dynamical variable turns the correction terms into a gravitational action. In other words, the regulated (but not renormalized) effective action of matter on a curved spacetime is identified as the gravitational action that determines the dynamics of spacetime geometry. In order to avoid curvature terms of arbitrarily high orders to be induced into the gravitational action, one usually assumes in Sakharov’s approach that quantum corrections beyond one-loop order are somehow suppressed. Recall that already the squared Riemann curvature terms, which are present in the one-loop effective action of matter, generate new massive degrees of freedom that carry negative energy [3, 2], so-called ghosts, which generate severe problems [4]. When coupled to ordinary fields, ghosts cause the system to evolve to an infinitely excited state without a change in total energy. The inclusion of higher-order curvature terms only makes the situation worse. Such terms are typically present in the effective field theory of gravity, which is obtained as a low-energy limit of various quantum theories. In the induced gravitational action obtained in Sec. 4, the extra degrees of freedom are found to have masses around the Planck mass, since the dimensionless coupling constants of the squared curvature terms are smaller than one, and hence the characteristic length scale in their Yukawa potentials [3, 2] is the Planck length. Thus, the effect of the extra degrees of freedom can be regarded to be negligible at low energies and long distances. We assume that a consistent description of quantum gravity and matter will eventually solve the ghost problem. Therefore, in this work, we adopt the view that higher-derivative contributions to the effective action can be ignored in the low-energy regime.

The most important lesson of Sakharov’s vision is that any fundamental theory that includes or produces a curved spacetime manifold, on which a quantum field theory of matter can be set up, necessarily produces gravity as well. In this sense, gravity is an unavoidable and necessary companion of quantum matter. The idea to induce gravity from quantum effects

and spontaneous symmetry breaking was further developed, particularly by Adler and Zee [5, 7, 6, 8, 9, 10, 11, 12]. A recent perspective on Sakharov's induced gravity is given in Ref. [13], which includes a discussion on different interpretations of quantum corrections. The idea of induced gravity was naturally expanded to the realm of quantum gravity, where it has been used in particular to derive Einstein gravity as a low-energy effective theory of scale-invariant (and asymptotically-free) quantum field theory of gravity [12, 14, 15, 16, 17].

When applied to matter fields, Sakharov's induced gravity is an early representative of the emergent approach to gravity, where gravity or at least gravitational dynamics is regarded to arise from a quantum theory that does involve a gravitational interaction in its initial definition. Sakharov's approach only addresses the induction of gravitational dynamics by producing the gravitational action, and consequently the field equations, but it does not produce spacetime, since the existence of a curved spacetime manifold is presumed. Since then many people have wondered whether spacetime too could be an emergent concept, and perhaps even gravity as a whole might have an emergent origin. An intriguing indication towards this view is the deep connection of gravity and thermodynamics. It all began from Bekenstein's discovery of the area law for black hole entropy [18, 19, 20, 21], which was predated by observations that the horizon surface area and the irreducible mass of black holes can never decrease in a classical process [22, 23, 24, 25]. The identification of horizon surface area as entropy and surface gravity as temperature (both up to a constant factor) quickly led to a full analogy between black hole mechanics and thermodynamics [26], and soon after to Hawking's discovery of thermal radiation of black holes [27]. The universal upper limit on the entropy that can be contained within a finite region of space which has a finite amount of energy, namely, the Bekenstein bound [28, 29, 30, 31] means that a physical system with a finite energy in a finite space is described by at most a certain finite amount of information. A covariant generalization of the Bekenstein entropy bound has been achieved [32], as well as a similar bound for asymptotically de Sitter spacetimes [33]. Black hole thermodynamics and the entropy bound were a major inspiration for 't Hooft's proposal of the holographic principle [34] and its subsequent string-theoretic interpretation by Susskind [35]. The gauge/gravity duality is the most rigorous realization of the holographic principle [36]. It has been argued that since the black hole entropy is, at least in part, an entanglement entropy [37], it would be most satisfactory if the gravitational action is induced as Sakharov proposed, so that all black hole entropy would be entanglement entropy [38, 39]. Black hole thermodynamics (and the holographic principle) has also had an influence on many other approaches to understand the relation between gravity and thermodynamics. The Einstein equation has been derived locally on Rindler causal horizons as a thermodynamic equation of state [40, 41]. An extension of black hole ther-

dynamics to causal horizons has been considered [42]. The holographic relation of bulk and surface terms in gravitational actions [43] has been used in arguing that the field equations of any diffeomorphism invariant theory of gravity have a thermodynamic reinterpretation, and showing that the equipartition of energy on the microscopic degrees of freedom of a horizon can be used to derive gravity [44, 45, 46]. It has even been proposed that gravity is an entropic force caused by changes in the information associated with the positions of material bodies [48]. The existence of gravitationally bound quantum states [49] can be used to impose some conditions on the fundamental microscopic theory behind entropic gravity [50, 51], which is presently unknown. These results suggest a view of gravity and spacetime as emergent concepts, which may have a thermodynamic origin.

The gauge theory approach to gravity has been highly influential ever since the gauge invariance idea introduced by Weyl [52] for $U(1)$ was generalized to $SU(2)$ by Yang and Mills [53] and to all semisimple Lie groups by Utiyama [54], who considered the gauging of the Lorentz group for the first time. The first consistent gauge theory of gravity was obtained by Kibble via gauging of the Poincaré group [55], the symmetry group of the Minkowski spacetime, which was used to derive the Einstein–Cartan–Sciama–Kibble theory of gravity [56, 55], but more generally yields a Lagrangian that includes quadratic curvature and torsion terms [57, 58]. From there on, gauge theories of every symmetry group related to gravity have been proposed and explored, including the group of translations [59], the Weyl group (Poincaré group plus scale transformations) [60], the conformal and superconformal groups [61], the affine group [62], and so on. See Ref. [63] for a review of various gauge theories of gravity. The gauge theory approach has also been used in attempts to understand the relation of gravity and quantum mechanics. For example, several proposals for the gauge theory of gravity on noncommutative spacetime have been considered, e.g. [65, 64, 66, 67].

In this work, we consider Sakharov’s induced gravity on a Riemann–Cartan spacetime with both curvature and torsion. We shall derive the induced gravitational action at one-loop order for free scalar and Dirac fields. The mass scale that determines the induced gravitational constants, especially the induced Newton constant G_{ind} , is the ultraviolet cutoff Λ for the effective action of matter fields. For gravity to have the observed strength, $G_{\text{ind}}^{-1} = 8\pi M_{\text{P}}^2$, the ultraviolet cutoff Λ has to be comparable to the Planck mass M_{P} . The effect of torsion is generally weak except when the density of matter and spin is very high [68, 69]. When the gravitational Lagrangian is the curvature scalar of the Riemann–Cartan spacetime, $\mathcal{L} = \frac{1}{2\kappa} \tilde{R}$, which gives the Einstein–Cartan–Sciama–Kibble theory of gravity, torsion does not propagate in vacuum. In more general theories, especially, in the generic Poincaré gauge theory of gravity (PG) with a Lagrangian that is quadratic in torsion and curvature, $\mathcal{L} = \tilde{R} + T^2 + \tilde{R}^2$, torsion does propagate in vacuum [70, 71, 72] (for recent development and further references, see [73]),

which may also improve the chances for its detection in the future. For a discussion of several physical implications of the torsion of spacetime, see Refs. [68, 69, 74]. The effect of the induced gravitational terms with dimensionless couplings, including the squared curvature terms, is found to be very small in the low-energy regime, since their induced coupling constants depend on the logarithm of the ultraviolet cutoff, which implies that those coupling constants are of the order of one or lower.

Including torsion into the gravitational theory is highly appealing, since then the intrinsic angular momentum of matter and gravity can be naturally incorporated into the theory. In general, both torsion and nonmetricity should be considered, in addition to curvature, in order to obtain a comprehensive understanding of the nature of induced gravity in the non-Riemannian setting. In this work, however, we confine the treatment to spacetimes with a vanishing nonmetricity. This limitation is motivated by our main goal, which is the emergence of PG as an induced theory of gravity via Sakharov’s mechanism. PG is a viable alternative to GR, which has been studied extensively [63]. We show that the quantization of Dirac fields on Riemann–Cartan spacetime induces the low-energy action of PG. The high-energy part of the induced action is found to contain additional terms compared to the action of PG. That is expectable, since the effective action is not limited to contain only terms quadratic in the field strengths, namely, in torsion and curvature. On a more general spacetime with a non-metric compatible connection, a general metric-affine gauge theory of gravity should be induced in the same way.

2 Sakharov’s approach

We generalize Sakharov’s approach to Riemann–Cartan spacetime. As in Riemannian spacetime, the procedure can be considered to consist of five steps:

1. Assume a Lorentzian spacetime manifold. The geometric notions of Riemann–Cartan spacetime can be derived by applying Einstein’s Equivalence Principle to a Dirac spinor [63], which can be considered to describe a neutron in a gravitational field, instead of applying it to a point mass as in GR.
2. Leave the dynamics of the geometry undetermined, i.e., consider it as an arbitrary classical background and do not define an action for gravity.
3. Quantize matter fields and determine their effective action. In the language of Feynman diagrams, the one-loop effective action represents the sum of all one-loop diagrams coupled to an arbitrary number of

external gravitons. In this visualization, gravitons refer to small perturbations of the geometric background fields around a fixed background. Regularize the effective action, and obtain the contribution of the background geometry to the action.

4. Elevate the geometric background fields to gravitational variables. We consider two possible choices for the variables: the vierbein and the spin connection (a first-order formalism), or the metric and the torsion (a second-order formalism).
5. Identify the regulated one-loop effective action as the leading contribution to gravity. The gravitational action induced at one-loop order consists of contributions to vacuum energy, curvature terms up to second order and torsion terms up to fourth order.

Sakharov's approach, and many subsequent induced gravity approaches, involve several problems:

1. The induced Newton constant is not guaranteed to be unique and positive. In general, the Feynman amplitudes for stress-energy tensor operators are complex, and should be continued analytically. This problem also appears in the approaches [8, 10] where gravity is induced via symmetry breaking [76]. Some ways to address the problem have been proposed [77]. A further ambiguity is caused by the choice of a regularization method, since different methods imply different quantum corrections.

An ultraviolet-finite induced Newton constant can be obtained by including both scalar fields and spin-1/2 fields and by imposing fine-tuning constraints [39] on the numbers, masses and couplings of the scalar and spin-1/2 fields. Unfortunately, the masses of the constituent fields have to be comparable with the Planck mass, which causes problems in the presence of gravity, since quantum gravity effects should become significant at high energies.

2. The introduction of dimensional parameters that determine the scale of gravitational couplings is not fully convincing. In our simplistic approach, cutoff regularization is used to set the mass parameter of the induced theory of gravity to be of the order of the Planck mass. Alternatively, one could use any other regularization method, for example, Pauli-Villars regulators with sufficiently high masses. If the scale of gravitational couplings is set by the masses of fundamental fields, one generally requires fields whose masses are comparable with the Planck, which is problematic.
3. The elevation of the geometric notions to gravitational variables after the quantization of matter fields has no physical motivation (other

than that it works). This problem is a consequence of the fact that Sakharov's approach does not address the emergence of spacetime geometry but rather only the emergence of gravitational dynamics.

3 Geometric definitions

The covariant derivatives associated with the connection involving torsion $\tilde{\Gamma}$ and the torsionless connection Γ are denoted by $\tilde{\nabla}$ and ∇ , respectively. The connections on Riemann–Cartan spacetime are defined to be metric compatible. Namely, the nonmetricity tensor is assumed to vanish throughout this work, $Q_{\mu\nu\rho} = \tilde{\nabla}_\mu g_{\nu\rho} = 0$. Relaxing the metric-compatibility would lead to more general spacetime geometries, e.g., Weyl–Cartan [60] or metric-affine [62, 78], which are not considered here. Greek indices (μ, ν, \dots) refer to a coordinate basis, and Latin indices (a, b, \dots) refer to an orthonormal noncoordinate basis.

The connection coefficient $\tilde{\Gamma}_{\mu\nu}{}^\rho$ can be written as the sum of the Christoffel symbol $\Gamma_{\mu\nu}{}^\rho$ and the contortion tensor $K_{\mu\nu}{}^\rho$,

$$\tilde{\Gamma}_{\mu\nu}{}^\rho = \Gamma_{\mu\nu}{}^\rho + K_{\mu\nu}{}^\rho, \quad (1)$$

where the contortion tensor $K_{\mu\nu}{}^\rho$ is defined by the torsion $T_{\mu\nu}{}^\rho$,

$$T_{\mu\nu}{}^\rho = 2\tilde{\Gamma}_{[\mu\nu]}{}^\rho = \tilde{\Gamma}_{\mu\nu}{}^\rho - \tilde{\Gamma}_{\nu\mu}{}^\rho, \quad (2)$$

as

$$K_{\mu\nu}{}^\rho = \frac{1}{2} (T_{\mu\nu}{}^\rho + T_{\mu\nu}{}^\rho + T_{\nu\mu}{}^\rho). \quad (3)$$

In the second-order formalism of gravity, the independent gravitational variables can be chosen as the metric and the torsion, which determine the connection (1). Torsion can be decomposed into three irreducible components: the trace vector $\mathcal{V}_\mu = T_{\mu\nu}{}^\nu$, the axial vector $\mathcal{A}^\mu = T_{\nu\rho\sigma}\epsilon^{\nu\rho\sigma\mu}$, and the tensor component $\mathcal{T}_{\mu\nu}{}^\rho$ with vanishing vector and axial vector parts, $\mathcal{T}_{\mu\nu}{}^\nu = 0$ and $\mathcal{T}_{\nu\rho\sigma}\epsilon^{\nu\rho\sigma\mu} = 0$. Then, in four-dimensional spacetime, we have

$$T_{\mu\nu\rho} = \frac{1}{3} (\mathcal{V}_\mu g_{\nu\rho} - \mathcal{V}_\nu g_{\mu\rho}) - \frac{1}{6} \epsilon_{\mu\nu\rho\sigma} \mathcal{A}^\sigma + \mathcal{T}_{\mu\nu\rho}. \quad (4)$$

The components of torsion couple to matter fields in different ways, which is discussed for scalar and Dirac fields in Sec. 4. The contortion tensor is expressed in terms of the components of torsion as

$$K_{\mu\nu}{}^\rho = \frac{1}{3} (g_{\mu\nu} \mathcal{V}^\rho - \delta_\mu{}^\rho \mathcal{V}_\nu) - \frac{1}{12} \epsilon_{\mu\nu\lambda\sigma} g^{\lambda\rho} \mathcal{A}^\sigma + \frac{1}{2} (\mathcal{T}_{\mu\nu}{}^\rho + \mathcal{T}_{\mu\nu}{}^\rho + \mathcal{T}_{\nu\mu}{}^\rho), \quad (5)$$

and we note that $K_{\mu\nu}{}^\mu = -K_\mu{}^\mu{}_\nu = -\mathcal{V}_\nu$ and $K_{\mu\nu\rho}\epsilon^{\mu\nu\rho\sigma} = \frac{1}{2}\mathcal{A}^\sigma$.

The curvature tensor of the connection (1) can be written in terms of the curvature tensor of the torsionless connection and the contortion tensor as¹

$$\tilde{R}_{\mu\nu}{}^\rho{}_\sigma = R_{\mu\nu}{}^\rho{}_\sigma + 2\nabla_{[\mu}K_{\nu]\sigma}{}^\rho + 2K_{[\mu|\lambda}{}^\rho K_{|\nu]\sigma}{}^\lambda, \quad (6)$$

where

$$\tilde{R}_{\mu\nu}{}^\rho{}_\sigma = 2\partial_{[\mu}\tilde{\Gamma}_{\nu]\sigma}{}^\rho + 2\tilde{\Gamma}_{[\mu|\lambda}{}^\rho\tilde{\Gamma}_{|\nu]\sigma}{}^\lambda, \quad (7)$$

and $R_{\mu\nu}{}^\rho{}_\sigma$ is defined similarly in terms of $\Gamma_{\mu\nu}{}^\rho$. We may also express the curvature tensor of the torsionless connection as

$$R_{\mu\nu}{}^\rho{}_\sigma = \tilde{R}_{\mu\nu}{}^\rho{}_\sigma - 2\tilde{\nabla}_{[\mu}K_{\nu]\sigma}{}^\rho - T_{\mu\nu}{}^\lambda K_{\lambda\sigma}{}^\rho + 2K_{[\mu|\lambda}{}^\rho K_{|\nu]\sigma}{}^\lambda, \quad (8)$$

where everything on the right-hand side is defined in terms of the connection with torsion. Note that the order of tensor indices on curvature and torsion is such that the commutator of two covariant derivatives is written as

$$[\tilde{\nabla}_\mu, \tilde{\nabla}_\nu]V^\rho = \tilde{R}_{\mu\nu}{}^\rho{}_\sigma V^\sigma - T_{\mu\nu}{}^\sigma \tilde{\nabla}_\sigma V^\rho, \quad (9)$$

and the order will persist when the vierbein formalism and the spin representation are considered. The relations between the Ricci tensors, $\tilde{R}_{\mu\nu} = \tilde{R}_{\mu\rho}{}^\rho{}_\nu$ and $R_{\mu\nu} = R_{\mu\rho}{}^\rho{}_\nu$, and the scalar curvatures, $\tilde{R} = g^{\mu\nu}\tilde{R}_{\mu\nu}$ and $R = g^{\mu\nu}R_{\mu\nu}$, can be obtained from the relations (6) and (8). For the scalar curvatures we get

$$\tilde{R} = R - 2\nabla_\mu \mathcal{V}^\mu + \frac{2}{3}\mathcal{V}_\mu \mathcal{V}^\mu - \frac{1}{24}\mathcal{A}_\mu \mathcal{A}^\mu - \frac{1}{2}\mathcal{T}_{\mu\nu\rho}\mathcal{T}^{\mu\nu\rho}, \quad (10)$$

or the other way around,

$$R = \tilde{R} + 2\tilde{\nabla}_\mu \mathcal{V}^\mu + \frac{4}{3}\mathcal{V}_\mu \mathcal{V}^\mu + \frac{1}{24}\mathcal{A}_\mu \mathcal{A}^\mu + \frac{1}{2}\mathcal{T}_{\mu\nu\rho}\mathcal{T}^{\mu\nu\rho}. \quad (11)$$

In the first-order formalism, which is particularly used in the gauge theory approach to gravity, the independent variables are the vierbein $e^a{}_\mu$ and the spin connection $\tilde{\omega}_\mu{}^a{}_b$, or the corresponding one-forms, the coframe $\theta^a = e^a{}_\mu dx^\mu$ and the connection $\tilde{\omega}^a{}_b = \tilde{\omega}_\mu{}^a{}_b dx^\mu$. These variables are the gauge fields that are required to ensure gauge invariance of the action. Additionally, in a metric-affine gauge theory like PG, we assume the presence of a metric, $g_{ab}\theta^a \otimes \theta^b$, which is here taken to be an orthonormal coframe, $g_{ab} = \eta_{ab}$ (the Minkowski metric), so that the metric in a coordinate basis is defined as

$$g_{\mu\nu} = \eta_{ab}e^a{}_\mu e^b{}_\nu. \quad (12)$$

We consider PG, which is based on the Lorentz connection, $\tilde{\omega}^{ab} = \tilde{\omega}^{[ab]}$, i.e., on an antisymmetric linear connection. More general gauge theories

¹Tensor indexes in brackets are antisymmetrized, $a_{[\mu}b_{\nu]} = \frac{1}{2}(a_\mu b_\nu - a_\nu b_\mu)$, and indexes between vertical lines are excluded from the antisymmetrization, $a_{[\mu|\rho}b_{|\nu]} = \frac{1}{2}(a_{\mu\rho}b_\nu - a_{\nu\rho}b_\mu)$.

of gravity require a more general linear connection.² In the presence of a metric, it is possible to decompose the spin connection into a torsionless connection and a contortion component

$$\tilde{\omega}_\mu{}^{ab} = \omega_\mu{}^{ab} + K_\mu{}^{\nu\rho} e_\nu^a e_\rho^b, \quad (13)$$

which consists of the torsionless part $\omega_\mu{}^{ab}$ and the part proportional to contortion. The spin connection is related to the connection in a coordinate frame by the so-called tetrad postulate

$$\tilde{\nabla}_\mu e_\nu^a = \partial_\mu e_\nu^a + \tilde{\omega}_\mu{}^a{}_b e_\nu^b - \tilde{\Gamma}_{\mu\nu}{}^\rho e_\rho^a = 0, \quad (14)$$

which can be equivalently written as

$$\tilde{\omega}_\mu{}^a{}_b = \left(\tilde{\Gamma}_{\mu\nu}{}^\rho e_\rho^a - \partial_\mu e_\nu^a \right) e_b{}^\nu. \quad (15)$$

The torsion two-form is defined in terms of e_μ^a and $\tilde{\omega}_\mu{}^a{}_b$ as the exterior covariant derivative of the orthonormal coframe,

$$T^a = d\theta^a + \tilde{\omega}^a{}_b \wedge \theta^b = \frac{1}{2} T_{\mu\nu}{}^a dx^\mu \wedge dx^\nu, \quad (16)$$

where the components are defined as

$$T_{\mu\nu}{}^a = 2\partial_{[\mu} e_{\nu]}^a + 2\tilde{\omega}_{[\mu}{}^a{}_b e_{\nu]}^b. \quad (17)$$

The curvature two-form is defined as

$$\tilde{R}^a{}_b = d\tilde{\omega}^a{}_b + \tilde{\omega}^a{}_c \wedge \tilde{\omega}^c{}_b = \frac{1}{2} \tilde{R}_{\mu\nu}{}^a{}_b dx^\mu \wedge dx^\nu, \quad (18)$$

where the components are written as

$$\tilde{R}_{\mu\nu}{}^a{}_b = 2\partial_{[\mu} \tilde{\omega}_{\nu]}{}^a{}_b + 2\tilde{\omega}_{[\mu}{}^a{}_c \tilde{\omega}_{\nu]}{}^c{}_b. \quad (19)$$

The curvature $R_{\mu\nu}{}^a{}_b$ for the torsionless connection $\omega_\mu{}^a{}_b$ is defined similarly as (19). In a coordinate basis, the components of the curvature are given as

$$\tilde{R}_{\mu\nu}{}^\rho{}_\sigma = \tilde{R}_{\mu\nu}{}^a{}_b e_a{}^\rho e^b{}_\sigma. \quad (20)$$

When expressed entirely in the orthonormal frame $\{e_a\}$ (and the coframe $\{\theta^a\}$), where $e_a = e_a{}^\mu \partial_\mu$, the components of the torsion and the curvature are given as

$$\tilde{R}_{ab}{}^c{}_d = 2e_{[a} \tilde{\omega}_{b]}{}^c{}_d + 2\tilde{\omega}_{[a}{}^c{}_e \tilde{\omega}_{b]}{}^e{}_d - c_{ab}{}^e \tilde{\omega}_e{}^c{}_d, \quad (21)$$

$$T_{ab}{}^c = 2\tilde{\omega}_{[a}{}^c{}_{b]} - c_{ab}{}^c, \quad (22)$$

where $\tilde{\omega}_a{}^c{}_b = e_a{}^\mu \tilde{\omega}_\mu{}^c{}_b$, and they involve the anholonomy of the basis:

$$[e_a, e_b] = c_{ab}{}^c e_c, \quad c_{ab}{}^c = 2e_{[a}{}^\mu \partial_\mu e_{b]}{}^\nu e^c{}_\nu. \quad (23)$$

²Extending the local Poincaré group with local scale transformations would require the connection to have a nonvanishing trace component, which leads to a gauge theory of gravity with Weyl–Cartan geometry [60]. A general metric-affine gauge theory of gravity is based on the local affine group, and it requires an unrestricted linear connection [62, 78].

4 Induced gravitational action from quantized matter fields

One can consider matter fields that are coupled to curvature (and in the present case also to torsion) minimally, although there is no physical principle that would require such a restriction. In some cases, e.g. for scalar fields, nonminimal couplings are necessary in order to achieve renormalizability. Here our goal is not renormalization, but rather derivation of the one-loop effective action that is regulated but not renormalized. That way the effective action can be regarded as the origin of gravity. Therefore, the presence of nonminimal couplings is not necessary for the present approach, but we shall consider them for the sake of generality.

4.1 Scalar fields

First consider a free real scalar field ϕ on four-dimensional Riemann–Cartan spacetime. We can obtain the minimally-coupled Lagrangian from a free-field Lagrangian on Minkowski spacetime with the replacement $\partial_\mu \rightarrow \tilde{\nabla}_\mu$. The usual free-field Lagrangian on Minkowski spacetime with the kinetic part $\frac{1}{2}\eta^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$ gives the same Lagrangian as in Riemannian spacetime, since $\tilde{\nabla}_\mu\phi = \nabla_\mu\phi = \partial_\mu\phi$, and hence the torsion does not appear in it. Expressing the kinetic part of the Lagrangian in Minkowski spacetime as $-\frac{1}{2}\phi\eta^{\mu\nu}\partial_\mu\partial_\nu\phi$ leads to a different Lagrangian with a coupling to the vector component of torsion as

$$-\frac{1}{2}\phi\tilde{\square}\phi = -\frac{1}{2}\phi(\square\phi + \mathcal{V}^\mu\nabla_\mu\phi), \quad (24)$$

where

$$\tilde{\square} = g^{\mu\nu}\tilde{\nabla}_\mu\tilde{\nabla}_\nu, \quad \square = g^{\mu\nu}\nabla_\mu\nabla_\nu. \quad (25)$$

Thus, the minimal replacement rule does not lead to a unique scalar field Lagrangian in Riemann–Cartan spacetime. Unlike in a Riemannian spacetime, the form of the initial Lagrangian in flat spacetime matters. Since it would be a limited viewpoint to consider only the coupling to the vector component of torsion, we will include other torsion terms as well. Hence, we consider the parity-conserving free field Lagrangian with all nonminimal coupling terms

$$\mathcal{L} = \frac{1}{2} \left(g^{\mu\nu}\nabla_\mu\phi\nabla_\nu\phi - m^2\phi^2 - \sum_{i=1}^5 \xi_i P_i \phi^2 \right), \quad (26)$$

where ξ_i are dimensionless coupling constants and the corresponding even-parity curvature and torsion terms are

$$P_1 = R, \quad P_2 = \nabla_\mu\mathcal{V}^\mu, \quad P_3 = \mathcal{V}_\mu\mathcal{V}^\mu, \quad P_4 = \mathcal{A}_\mu\mathcal{A}^\mu, \quad P_5 = \mathcal{T}_{\mu\nu\rho}\mathcal{T}^{\mu\nu\rho}. \quad (27)$$

The Lagrangian (26) could alternatively be written in terms of $\tilde{\nabla}$, \tilde{R} and the components of torsion.

Note that we choose to write the Lagrangian in terms of the torsionless covariant derivative and its curvature, since it makes the calculations more convenient by enabling the use of techniques developed for quantum fields on Riemannian spacetime (see the monographs [79, 80]). That is, the metric and the torsion are regarded as the background fields. Note, however, that the gravitational variables are not chosen yet. They will be chosen later, after the quantum effective action has been obtained. Similar to the case of spacetime without torsion, where only the first nonminimal coupling $\xi R\phi^2$ appears, the nonminimal couplings would be necessary for renormalization [81]. Naturally, including pseudoscalar fields and/or complex scalar fields would allow further nonminimal coupling terms, which would again be necessary for renormalization [81]. Recall, however, that our present goal is not renormalization.

The action is defined by the Lagrangian (26) as

$$S = \int d^4x \sqrt{-g} \mathcal{L} = -\frac{1}{2} \int d^4x \sqrt{-g} \phi D\phi, \quad (28)$$

where

$$D = \square + m^2 + \sum_{i=1}^5 \xi_i P_i. \quad (29)$$

The one-loop effective action is defined [82, 80] in terms of D as

$$S_{\text{eff}}^{(1)} = \frac{i}{2} \ln \det (l^2 D) = \frac{i}{2} \text{Tr} \ln (l^2 D), \quad (30)$$

where the parameter l with dimension of length was introduced for dimensional reasons, so that $l^2 D$ is dimensionless. Since we consider only one-loop corrections, and hence there is no need to keep track of higher powers of \hbar , we have set $\hbar = 1$, along with $c = 1$. We can use the identity

$$\ln \left(\frac{D}{D_0} \right) = - \int_0^\infty \frac{d\tau}{\tau} [\exp(-i\tau D) - \exp(-i\tau D_0)], \quad (31)$$

where D_0 is the operator for a suitable reference background $(g_0, \tilde{\Gamma}_0)$,³ in order to obtain

$$S_{\text{eff}}^{(1)} = S_{\text{eff},0}^{(1)} - \frac{i}{2} \text{Tr} \int_0^\infty \frac{d\tau}{\tau} [\exp(-i\tau D) - \exp(-i\tau D_0)], \quad (32)$$

where $S_{\text{eff},0}^{(1)}$ is the one-loop effective action for the reference background. On a noncompact spacetime the reference background has to be chosen so that

³Both operators D and D_0 are considered to have a small negative imaginary part in order to avoid divergence of the integration in (31).

the physical action for induced gravity $S_{\text{phys}}^{(1)} = S_{\text{eff}}^{(1)} - S_{\text{eff},0}^{(1)}$ is well defined. We shall consider a compact spacetime for simplicity. This does not limit the generality of the derivation. The contribution of a reference background can be easily included into the induced action afterwards, in case one needs the result for a noncompact spacetime. Thence, instead of associating the operator D_0 with a reference background, we can freely choose it to be proportional to an identity operator as $D_0 = l^{-2}I$, so that we obtain

$$S_{\text{eff}}^{(1)} = -\frac{i}{2} \text{Tr} \int_0^\infty \frac{d\tau}{\tau} \exp(-i\tau D) + \frac{i}{2} \lim_{\epsilon \rightarrow 0^+} \int_0^\infty \frac{ds}{s} \exp(-is(1-i\epsilon)) \text{Tr} I. \quad (33)$$

In units of mass, the dimensions of the above constant l and the integration variables τ and $s = l^{-2}\tau$ are $[l] = -1$, $[\tau] = -2$ and $[s] = 0$. The second term in (33) is a constant, which is irrelevant dynamically, since it does not depend on D or on any geometric quantities. Therefore, in the following, we drop the constant term and write

$$S_{\text{eff}}^{(1)} = -\frac{i}{2} \text{Tr} \int_0^\infty \frac{d\tau}{\tau} \exp(-i\tau D). \quad (34)$$

We express the operator in the effective action in terms of a kernel function $K(\tau; x, y)$, which is defined as

$$\exp(-i\tau D)\phi(x) = \int d^4y \sqrt{-g} K(\tau; x, y) \phi(y). \quad (35)$$

The kernel satisfies a Schrödinger-like equation

$$i \frac{d}{d\tau} K(\tau; x, y) = D K(\tau; x, y), \quad (36)$$

with the boundary condition

$$K(\tau = 0; x, y) = I \delta(x, y), \quad (37)$$

where I is the identity operator/matrix for the fields or field components. We are working with a spacetime of Lorentzian signature so that the kernel corresponds to a heat kernel with imaginary time [82, 83]. Strictly speaking, the heat kernel technique is mathematically well defined only in Euclidean signature, when the squared distance $(x - y)^2$ of points is positive definite. The kernel discussed here should be regarded as an analytic continuation of the heat kernel, and it is only useful for finding the local contributions (in the limit $y \rightarrow x$) to the effective action. The trace of the operator in (34) is given by

$$\text{Tr} \exp(-i\tau D) = \int d^4x \sqrt{-g} \text{tr} K(\tau; x, x), \quad (38)$$

where in the right-hand side the trace is taken over the field degrees of freedom.⁴ The (divergent) gravitational terms that we are interested in come from the zero end of the integral over τ . We use the local series expansion of the kernel [79, 82, 80, 83]

$$K(\tau; x, x) = \frac{i}{(4\pi i\tau)^2} \sum_{n=0}^{\infty} (i\tau)^n A_n(x), \quad (39)$$

where $A_n(x)$ are constructed from the geometric quantities and parameters involved in D , i.e., from the covariant derivative, the curvature, the torsion, and the masses and couplings of the fields. We assume that the spacetime has no boundary. If the spacetime had a boundary, we would have to include boundary terms into the series expansion of the operator (38). In the presence of boundaries, each term in the series expansion except the zeroth term involves an additional boundary contribution [83], and the series expansion (39) also involves half-integer terms, $n = \frac{1}{2}, \frac{3}{2}, \dots$, which are purely boundary terms.

Since the operator (29) is readily in the Laplace form

$$D = \square + B, \quad (40)$$

where the (endomorphism) term B does not involve derivatives,⁵ one obtains the first three terms of the kernel (39) as⁶

$$A_0 = I, \quad (41)$$

$$A_1 = \frac{1}{6}RI - B, \quad (42)$$

$$A_2 = \left(\frac{1}{180}R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - \frac{1}{180}R^{\mu\nu}R_{\mu\nu} + \frac{1}{72}R^2 - \frac{1}{30}\square R \right) I \\ + \frac{1}{2}B^2 - \frac{1}{6}RB + \frac{1}{6}\square B + \frac{1}{12}W^{\mu\nu}W_{\mu\nu}, \quad (43)$$

where

$$W_{\mu\nu} = [\nabla_\mu, \nabla_\nu]. \quad (44)$$

For a single real scalar field, the identity is of course one-dimensional, $I = 1$, (44) vanishes, $W_{\mu\nu} = 0$, and

$$B = m^2 + \sum_{i=1}^5 \xi_i P_i. \quad (45)$$

⁴When a field with several components or several fields are involved, the operator D and its kernel K are matrix-valued.

⁵We mean that B does not involve derivative operators acting on the field. Of course, B itself involves derivatives of the gravitational fields, namely, in the present case the curvature and torsion terms in (45).

⁶See Refs.[82, 85] and references therein for different techniques of derivation for the kernel coefficients, and e.g. Refs.[79, 80, 83] for its use.

Which terms of the expansion of the kernel (39) appear in the regularized one-loop effective action depends on the chosen regularization method. For example, dimensional regularization involves only the term A_2 , which contains quadratic curvature and torsion terms. In that sense, it is a too powerful regularization method for our purposes. We choose to use the cutoff regularization, since it involves all the given terms (41)–(43). The lower limit of the integral over τ is cut off at Λ^{-2} for the three divergent terms, where the ultraviolet cutoff parameter Λ has the dimension of mass, $[\Lambda] = 1$. For the third term with A_2 the upper limit of the integral is cut off at $\tau_0 = \epsilon^{-2}$, where ϵ is an infrared cutoff. We obtain the kernel expansion of the cutoff-regularized one-loop effective action as

$$\begin{aligned} S_{\text{eff}}^{(1)} &= \frac{1}{32\pi^2} \sum_{n=0}^{\infty} \int_{0 \text{ or } \Lambda^{-2}}^{\infty \text{ or } \epsilon^{-2}} d(i\tau)(i\tau)^{n-3} \int d^4x \sqrt{-g} \text{tr } A_n(x) \\ &= \frac{\Lambda^4}{64\pi^2} \int d^4x \sqrt{-g} \text{tr } A_0 + \frac{\Lambda^2}{32\pi^2} \int d^4x \sqrt{-g} \text{tr } A_1 \\ &\quad + \frac{\ln(\Lambda/\epsilon)}{16\pi^2} \int d^4x \sqrt{-g} \text{tr } A_2 + \text{ultraviolet-finite terms.} \end{aligned} \quad (46)$$

The term of order Λ^4 , i.e., the quartic divergence, is given as $\text{tr } A_0 = 1$, which contributes only to the vacuum energy. The term of order Λ^2 in the one-loop effective action (46) is given as

$$\text{tr } A_1 = -m^2 + \left(\frac{1}{6} - \xi_1\right) R - \sum_{i=2}^5 \xi_i P_i. \quad (47)$$

The term proportional to the logarithm of the cutoff contains the second-order gravitational terms: quadratic curvature terms and torsion terms up to fourth power. The term $\frac{1}{2}B^2$ of the contribution (43) proportional to $\ln(\Lambda/\epsilon)$ also contains the first-order terms $m^2 \sum_{i=1}^5 \xi_i P_i$. In total, we have

$$\begin{aligned} \text{tr } A_2 &= \frac{1}{180} R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - \frac{1}{180} R^{\mu\nu} R_{\mu\nu} + \frac{1}{72} R^2 - \frac{1}{30} \square R \\ &\quad + \frac{1}{2} \left(m^2 + \sum_{i=1}^5 \xi_i P_i\right)^2 - \frac{1}{6} R \left(m^2 + \sum_{i=1}^5 \xi_i P_i\right) + \frac{1}{6} \sum_{i=1}^5 \xi_i \square P_i \\ &= \frac{1}{180} R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - \frac{1}{180} R^{\mu\nu} R_{\mu\nu} + \frac{1}{2} \left(\frac{1}{36} - \frac{\xi_1}{3} + \xi_1^2\right) R^2 \\ &\quad + \frac{1}{6} \left(\xi_1 - \frac{1}{5}\right) \square R + \left(\xi_1 - \frac{1}{6}\right) R \sum_{i=2}^5 \xi_i P_i + \frac{1}{2} \left(\sum_{i=2}^5 \xi_i P_i\right)^2 \\ &\quad + \frac{1}{6} \sum_{i=2}^5 \xi_i \square P_i + \left(\xi_1 - \frac{1}{6}\right) m^2 R + m^2 \sum_{i=2}^5 \xi_i P_i + \frac{1}{2} m^4, \end{aligned} \quad (48)$$

where the sums in the latter expression are taken over the torsion terms, P_i with $i = 2, \dots, 5$.

The Gauss-Bonnet-Chern term

$$G = R_{\alpha\beta\gamma\delta} R_{\mu\nu\rho\sigma} \epsilon^{\alpha\beta\mu\nu} \epsilon^{\gamma\delta\rho\sigma} = R^{\mu\nu\rho\sigma} R_{\mu\nu\rho\sigma} - 4R^{\mu\nu} R_{\mu\nu} + R^2, \quad (49)$$

whose integral is a topological invariant in four-dimensional spacetime, can be used to absorb the Riemann tensor squared term from the effective action.

Alternatively, we could write the effective action in terms of the connection $\tilde{\nabla}$ with torsion and its curvature (7) by using the relations between the two connections (1) and their curvature tensors (8). That would be the appropriate way, if one chooses the first-order formalism, where $e^a{}_\mu$ and $\tilde{\omega}^a{}_\mu{}^b$ are the independent variables of the induced gravitational action. Such a case is considered in Sec. 5, where the induced action for PG is obtained.

4.2 Spin-1/2 fields

Next we consider a Dirac spinor field ψ . The Dirac field can couple to the vector and the axial vector components of torsion. Naturally, these couplings are of the same form as the couplings for any other vector and axial vector fields. In particular, the vector component \mathcal{V}_μ couples to the Dirac spinor in the same way as the electromagnetic field. In the Dirac action, spin-1/2 fields do not couple to curvature or to higher-order torsion invariants (27) due to dimensional reasons, since the mass dimensions are $[\psi] = \frac{3}{2}$, $[Riemann] = 2$ and $[Torsion] = 1$.⁷

The Hermitian Lagrangian for a free minimally coupled Dirac field is written as

$$\mathcal{L}_{\text{Dirac}}^{\text{min.}} = \frac{i}{2} \left(\bar{\psi} \gamma^\mu \tilde{\nabla}_\mu \psi - \tilde{\nabla}_\mu \bar{\psi} \gamma^\mu \psi \right) - m \bar{\psi} \psi, \quad (50)$$

where the spacetime-dependent γ matrices are defined as $\gamma^\mu = e_a{}^\mu \gamma^a$, where $e_a{}^\mu$ is the inverse of the vierbein $e^a{}_\mu$. In four-dimensional spacetime, the constant γ matrices satisfy

$$\{\gamma^a, \gamma^b\} = 2\eta^{ab} I, \quad (\gamma^0)^2 = I, \quad (\gamma^i)^2 = -I, \quad (\gamma^a)^\dagger = \gamma^0 \gamma^a \gamma^0, \quad (51)$$

where $a = 0, 1, 2, 3$, $i = 1, 2, 3$, $(\eta^{ab}) = \text{diag}(1, -1, -1, -1)$ and I is the four-dimensional identity matrix. The fifth γ matrix is defined as

$$\gamma^5 = i\gamma^0\gamma^1\gamma^2\gamma^3, \quad (52)$$

and it satisfies

$$(\gamma^5)^2 = I, \quad (\gamma^5)^\dagger = \gamma^5, \quad \{\gamma^5, \gamma^a\} = 0. \quad (53)$$

⁷We do not consider coupling constants with negative mass dimensions or couplings to fractional or negative powers of curvature and torsion.

The covariant derivative for the spinor and its conjugate $\bar{\psi} = \psi^\dagger \gamma^0$ is defined by

$$\begin{aligned}\tilde{\nabla}_\mu \psi &= \partial_\mu \psi + \frac{i}{2} \tilde{\omega}_\mu{}^{ab} \Sigma_{ab} \psi, \\ \tilde{\nabla}_\mu \bar{\psi} &= \partial_\mu \bar{\psi} - \frac{i}{2} \bar{\psi} \tilde{\omega}_\mu{}^{ab} \Sigma_{ab},\end{aligned}\tag{54}$$

where $\Sigma_{ab} = -\frac{i}{4}[\gamma_a, \gamma_b]$ satisfies the Lie algebra of $SO(1, 3)$. Likewise, the covariant derivative without torsion is

$$\begin{aligned}\nabla_\mu \psi &= \partial_\mu \psi + \frac{i}{2} \omega_\mu{}^{ab} \Sigma_{ab} \psi, \\ \nabla_\mu \bar{\psi} &= \partial_\mu \bar{\psi} - \frac{i}{2} \bar{\psi} \omega_\mu{}^{ab} \Sigma_{ab}.\end{aligned}\tag{55}$$

The action for the minimally coupled Lagrangian (50) can be written in two forms, either in terms of $\tilde{\nabla}$ or ∇ , as

$$\begin{aligned}S_{\text{Dirac}}^{\text{min.}} &= \int d^4x \sqrt{-g} \bar{\psi} \left(i \gamma^\mu \tilde{\nabla}_\mu - \frac{i}{2} \gamma^\mu \mathcal{V}_\mu - m \right) \psi \\ &= \int d^4x \sqrt{-g} \bar{\psi} \left(i \gamma^\mu \nabla_\mu + \frac{1}{8} \gamma^\mu \gamma^5 \mathcal{A}_\mu - m \right) \psi,\end{aligned}\tag{56}$$

where the boundary surface term coming from an integration by parts is assumed to vanish, $\int d^4x \partial_\mu (\sqrt{-g} \bar{\psi} \gamma^\mu \psi) = 0$. According to the second expression of (56) a minimally coupled spinor couples only to the axial vector component of torsion. We shall consider a more general Dirac field that couples to both the vector and axial components of torsion.

Written in terms of the torsionless covariant derivative, a nonminimally coupled Dirac field has the action

$$\begin{aligned}S_{\text{Dirac}}^{\text{non-min.}} &= \int d^4x \sqrt{-g} \bar{\psi} D_{1/2} \psi, \\ D_{1/2} &= i \gamma^\mu \mathcal{D}_\mu - m, \\ \mathcal{D}_\mu &= \nabla_\mu + i \alpha_1 \mathcal{V}_\mu + i \alpha_2 \gamma^5 \mathcal{A}_\mu,\end{aligned}\tag{57}$$

where α_1 and α_2 are dimensionless coupling constants for the vector and the axial vector components of torsion, respectively. The minimally coupled case (50) corresponds to the couplings $\alpha_1 = 0$, $\alpha_2 = -\frac{1}{8}$.

The one-loop effective action is defined as

$$S_{\text{eff}}^{(1)} = -i \ln \det(l D_{1/2}),\tag{58}$$

where the minus sign and the factor of two compared to the spin-0 case (34) come from the anticommuting and complex-valued nature of the Dirac field, respectively. Since the Dirac operator $D_{1/2}$ is a first-order differential

operator, we cannot use the technique of Sec. 4.1 directly. However, we can square the operator $D_{1/2}$ as follows (for a more detailed proof, see [89]). First we define a modification of the operator $D_{1/2}$ as

$$D_{1/2}^* = -i\gamma^\mu \mathcal{D}_\mu - m. \quad (59)$$

In even-dimensional spacetime, the matrix γ^5 , which is Hermitian and satisfies $(\gamma^5)^2 = I$, anticommutes with γ^μ . Hence, $\gamma^5 D_{1/2}^* \gamma^5 = D_{1/2}$,⁸ and we obtain

$$\det(lD_{1/2}^*) = \det(l\gamma^5 D_{1/2}^* \gamma^5) = \det(lD_{1/2}). \quad (60)$$

Therefore, the operator $D_{1/2}$ in the effective action (58) can be squared as

$$S_{\text{eff}}^{(1)} = -\frac{i}{2} \ln [\det(lD_{1/2})]^2 = -\frac{i}{2} \ln \det(l^2 D_{1/2} D_{1/2}^*). \quad (61)$$

Hence, the operator D in the effective action of a Dirac spinor is written as

$$D \equiv D_{1/2} D_{1/2}^* = (\gamma^\mu \mathcal{D}_\mu)^2 + m^2. \quad (62)$$

The operator D in (62) is not of the Laplace type for any of the derivatives \mathcal{D} , ∇ or $\hat{\nabla}$. We can see this by expanding the first term in (62) as

$$(\gamma^\mu \mathcal{D}_\mu)^2 = g^{\mu\nu} \mathcal{D}_\mu \mathcal{D}_\nu - 2i\alpha_2 \gamma^\mu \gamma^\nu \gamma^5 \mathcal{A}_\mu \mathcal{D}_\nu + \frac{1}{2} \gamma^\mu \gamma^\nu [\mathcal{D}_\mu, \mathcal{D}_\nu], \quad (63)$$

where the last term can be written as

$$\frac{1}{2} \gamma^\mu \gamma^\nu [\mathcal{D}_\mu, \mathcal{D}_\nu] = \frac{1}{4} R + \frac{i}{4} \alpha_1 [\gamma^\mu, \gamma^\nu] V_{\mu\nu} + \frac{i}{4} \alpha_2 [\gamma^\mu, \gamma^\nu] \gamma^5 A_{\mu\nu}, \quad (64)$$

which involves the following second-rank tensors⁹

$$\begin{aligned} V_{\mu\nu} &= \nabla_\mu \mathcal{V}_\nu - \nabla_\nu \mathcal{V}_\mu, \\ A_{\mu\nu} &= \nabla_\mu \mathcal{A}_\nu - \nabla_\nu \mathcal{A}_\mu. \end{aligned} \quad (65)$$

Therefore, the operator (62) contains a first-order derivative term. However, for any partial differential operator D that contains first-order and second-order partial derivatives, with the second-order term taking the form $g^{\mu\nu} \partial_\mu \partial_\nu$, there exists [85] a unique connection $\hat{\nabla}$ and a unique endomorphism B for which the operator takes the Laplace form

$$D = \hat{\square} + B, \quad \hat{\square} = g^{\mu\nu} \hat{\nabla}_\mu \hat{\nabla}_\nu. \quad (66)$$

⁸In even-dimensional spacetime, the matrices $\{\gamma^\mu\}$ and $\{-\gamma^\mu\}$ form equivalent representations of the Clifford algebra (see e.g. [80]).

⁹In a more general setting, the vector \mathcal{V}_μ and the axial vector \mathcal{A}_μ could contain not only the vector components of torsion but also gauge fields in some representation of a given gauge group. In that case, $V_{\mu\nu}$ and $A_{\mu\nu}$ would be extended to include the field strength tensors of the gauge fields.

That connection consists of the Riemannian connection and a gauge (bundle) part. In the present case (62), the operator (66) is given by

$$\begin{aligned}\hat{\nabla}_\mu &= \mathcal{D}_\mu - i\alpha_2 \underline{\gamma}^\nu \underline{\gamma}_\mu \gamma^5 \mathcal{A}_\nu \\ &= \nabla_\mu + i\alpha_1 \mathcal{V}_\mu + \frac{i}{2} \alpha_2 [\underline{\gamma}_\mu, \underline{\gamma}_\nu] \gamma^5 \mathcal{A}^\nu\end{aligned}\quad (67)$$

and

$$B = \left(\frac{1}{4} R - 2\alpha_2^2 \mathcal{A}_\mu \mathcal{A}^\mu + m^2 \right) I + \frac{i}{4} \alpha_1 [\underline{\gamma}^\mu, \underline{\gamma}^\nu] V_{\mu\nu} + i\alpha_2 \gamma^5 \nabla_\mu \mathcal{A}^\mu. \quad (68)$$

The tensor $W_{\mu\nu}$ for the connection $\hat{\nabla}$ is given by

$$\begin{aligned}W_{\mu\nu} &= -\frac{1}{8} R_{\mu\nu\rho\sigma} [\underline{\gamma}^\rho, \underline{\gamma}^\sigma] + i\alpha_1 V_{\mu\nu} + i\alpha_2 \gamma^5 A_{\mu\nu} \\ &\quad + i\alpha_2 \gamma^5 \underline{\gamma}_\rho \left(\underline{\gamma}_\mu \nabla_\nu \mathcal{A}^\rho - \underline{\gamma}_\nu \nabla_\mu \mathcal{A}^\rho \right) \\ &\quad - \alpha_2^2 \left(\underline{\gamma}_\mu \underline{\gamma}_\rho \underline{\gamma}_\nu \underline{\gamma}_\sigma - \underline{\gamma}_\nu \underline{\gamma}_\rho \underline{\gamma}_\mu \underline{\gamma}_\sigma \right) \mathcal{A}^\rho \mathcal{A}^\sigma.\end{aligned}\quad (69)$$

The kernel expansion and regularization of the effective action are performed in a way identical to that of the scalar field case. Only now the operator D , the kernel K , the identity I and the tensor $W_{\mu\nu}$ are four-dimensional square matrices. The tensor $W_{\mu\nu}$ is given for the spinor in (69). The cutoff-regularized effective action has the same expression as for a scalar field (46) but with an opposite sign,

$$\begin{aligned}S_{\text{eff}}^{(1)} &= -\frac{\Lambda^4}{64\pi^2} \int d^4x \sqrt{-g} \text{tr} A_0 - \frac{\Lambda^2}{32\pi^2} \int d^4x \sqrt{-g} \text{tr} A_1 \\ &\quad - \frac{\ln(\Lambda/\epsilon)}{16\pi^2} \int d^4x \sqrt{-g} \text{tr} A_2 + \text{ultraviolet-finite terms}.\end{aligned}\quad (70)$$

Then we evaluate the traces of the terms (41)–(43) of the kernel expansion with (68) and (69). The first two terms of the effective action (70) are given by

$$\text{tr} A_0 = 4, \quad (71)$$

$$\text{tr} A_1 = -\frac{1}{3} R + 8\alpha_2^2 \mathcal{A}_\mu \mathcal{A}^\mu - 4m^2, \quad (72)$$

using $\text{tr} I = 4$ and $\text{tr} B = R - 8\alpha_2^2 \mathcal{A}_\mu \mathcal{A}^\mu + 4m^2$. Finally, after some lengthy algebra, we obtain the term proportional to $\ln(\Lambda/\epsilon)$ in the effective action

(70) as

$$\begin{aligned}
\text{tr } A_2 = & -\frac{7}{360}R^{\mu\nu\rho\sigma}R_{\mu\nu\rho\sigma} - \frac{1}{45}R^{\mu\nu}R_{\mu\nu} + \frac{1}{72}R^2 + \frac{1}{30}\square R \\
& + \frac{2}{3}\alpha_1^2 V^{\mu\nu}V_{\mu\nu} + \frac{2}{3}\alpha_2^2 A^{\mu\nu}A_{\mu\nu} - \frac{4}{3}\alpha_2^2 \square(\mathcal{A}_\mu \mathcal{A}^\mu) \\
& + \frac{4}{3}\alpha_2^2 \nabla_\mu (\mathcal{A}^\nu \nabla_\nu \mathcal{A}^\mu - \mathcal{A}^\mu \nabla_\nu \mathcal{A}^\nu) \\
& + \frac{1}{3}m^2 R - 8\alpha_2^2 m^2 \mathcal{A}_\mu \mathcal{A}^\mu + 2m^4.
\end{aligned} \tag{73}$$

This term does not explicitly contain terms involving both curvature and torsion. Note, however, that the term $\nabla_\mu (\mathcal{A}^\nu \nabla_\nu \mathcal{A}^\mu - \mathcal{A}^\mu \nabla_\nu \mathcal{A}^\nu)$ contains such a cross term between the Ricci tensor and the axial vector component of torsion, $R_{\mu\nu} \mathcal{A}^\mu \mathcal{A}^\nu$.

It is interesting to compare the torsion terms of the one-loop effective actions for a scalar field and a Dirac field. The effective Lagrangian for a scalar field (46) contains the following torsion terms:

$$\begin{aligned}
P_i, \quad \square P_i, \quad (i = 2, \dots, 5) \\
P_i P_j, \quad (i = 1, \dots, 5, j = 2, \dots, 5)
\end{aligned} \tag{74}$$

where P_i is defined in (27). The first term in (74) appears both as $\Lambda^2 \xi_i P_i$ and $(\ln \Lambda/\epsilon) m^2 \xi_i P_i$. The latter two terms in (74) appear in the term that is proportional to $\ln(\Lambda/\epsilon)$ in the effective action (46). On the other hand, the effective Lagrangian for a Dirac field (70) involves the following torsion terms:

$$\begin{aligned}
\mathcal{A}_\mu \mathcal{A}^\mu, \quad \square(\mathcal{A}_\mu \mathcal{A}^\mu), \quad V^{\mu\nu}V_{\mu\nu}, \quad A^{\mu\nu}A_{\mu\nu}, \\
\nabla_\mu \mathcal{A}^\nu \nabla_\nu \mathcal{A}^\mu, \quad (\nabla_\mu \mathcal{A}^\mu)^2, \quad R_{\mu\nu} \mathcal{A}^\mu \mathcal{A}^\nu.
\end{aligned} \tag{75}$$

There are no cross terms between the three components of torsion in the Dirac field case. It is noteworthy that the only common torsion terms in the effective Lagrangians for a scalar field and a Dirac field are the following three terms involving the squared norm $\mathcal{A}_\mu \mathcal{A}^\mu$ of the axial component of torsion:

$$\Lambda^2 \mathcal{A}_\mu \mathcal{A}^\mu, \quad (\ln \Lambda/\epsilon) m^2 \mathcal{A}_\mu \mathcal{A}^\mu, \quad (\ln \Lambda/\epsilon) \square(\mathcal{A}_\mu \mathcal{A}^\mu). \tag{76}$$

Notice that the quartic term $(\ln \Lambda/\epsilon) (\mathcal{A}_\mu \mathcal{A}^\mu)^2$ does not appear in the Dirac field case (73). All the rest of the torsion terms in (74) and (75) appear only for either a scalar field or for a Dirac field.

4.3 Induced gravitational couplings

We compare the induced gravitational actions (46) and (70) with the gravitational action

$$S = \frac{1}{16\pi G} \int d^4x \sqrt{-g} (R - 2\lambda + \dots), \tag{77}$$

where the dots stand for all possible generally covariant terms constructed from the torsion and the curvature (except R). The induced cosmological constant is obtained from the induced actions (46) and (70) as

$$\frac{\lambda_{\text{ind}}}{G_{\text{ind}}} = \sum_f \frac{C_f^{(0)}}{4\pi} \left(\frac{\Lambda^4}{2} - \Lambda^2 m_f^2 + \ln \left(\frac{\Lambda}{\epsilon} \right) m_f^4 \right), \quad (78)$$

where the sum is taken over all scalar and Dirac fields ($f = s, d$), and the constant $C_s^{(0)} = -1$ for a scalar field and $C_d^{(0)} = 4$ for a Dirac field, and the induced Newton constant G_{ind} will be discussed below. Since the masses m_f are taken to be much lighter than the ultraviolet cutoff, $m_f \ll \Lambda$, the dominant contribution to the cosmological constant is

$$\frac{\lambda_{\text{ind}}}{G_{\text{ind}}} \approx \frac{(4N_d - N_s)}{8\pi} \Lambda^4, \quad (79)$$

where N_d and N_s are the number of Dirac fields and the number of scalar fields, respectively. Since the ultraviolet cutoff has to be at least above the electroweak scale, $\Lambda \gtrsim 1 \text{ TeV}$, up to where the Standard Model has been tested accurately, and setting G_{ind} to its observed or of magnitude, $G_{\text{ind}} \sim 10^{-1} M_{\text{P}}^{-2}$, we obtain $\lambda_{\text{ind}} \lesssim -10^{-64} M_{\text{P}}^2$ for a scalar field. A Dirac spinor produces a positive cosmological constant, $\lambda_{\text{ind}} \gtrsim 10^{-64} M_{\text{P}}^2$. As usual, the vacuum energy obtained from a quantum field theory is far too high compared to the minuscule observed value $\lambda \sim 10^{-122} M_{\text{P}}^2$. Actually, we will soon see that the ultraviolet cutoff is expected to be comparable to the Planck mass in this approach, so that the prediction for λ_{ind} with fermionic matter is much higher than the above estimate, around $\lambda_{\text{ind}} \sim M_{\text{P}}^2$. Sakharov's approach does not help us with the vacuum energy problem. Consequently, the vacuum energy term that is proportional to the volume of spacetime is usually ignored in Sakharov's approach. It might, however, be possible to resolve the problem with fine tuning and radiative instability of the cosmological constant. For this purpose, we would like to highlight the proposal of vacuum energy sequestering [86], and in particular its recent local formulation [87, 88], where the perturbative instability of the vacuum energy contribution of quantized matter fields is tamed by letting the gravitational and cosmological constants become variables and introducing auxiliary volume four-forms for fixing the values of the given constants. Large contributions to the vacuum energy from the matter sector are cancelled by the sequestering mechanism. A potential problem with including the vacuum energy sequestering mechanism into Sakharov's induced gravity is that the gravitational terms (and consequently the gravitational and cosmological constants) are absent in the classical action prior to quantization of matter. Hence, the additional Lagrangian that is needed for the local sequestering mechanism has to be introduced by hand, unless one finds a way to induce

those terms along with the gravitational action. A possible clue for such a direction might be that the gravitational action along with the vacuum energy sequestering mechanism can be considered to emerge from gauge fixing in an underlying theory [88].¹⁰

Thus, the induced Newton constant is obtained as

$$\frac{1}{G_{\text{ind}}} = \sum_f \frac{C_f^{(1)}}{\pi} \left(\frac{\Lambda^2}{2} - \ln \left(\frac{\Lambda}{\epsilon} \right) m_f^2 \right), \quad (80)$$

where $C_s^{(1)} = \frac{1}{6} - \xi_{1s}$ for a scalar field and $C_d^{(1)} = \frac{1}{3}$ for a Dirac spinor. Since $m_f \ll \Lambda$, the dominant contribution to the Newton constant is obtained as

$$\frac{1}{G_{\text{ind}}} \approx \frac{(2N_d + N_s - 6 \sum_s \xi_{1s})}{12\pi} \Lambda^2. \quad (81)$$

This implies that for a single Dirac field $\Lambda \sim 10M_{\text{P}}$. Including a scalar field or scalar fields with a large negative coupling $\xi_{1s} \ll -1$ would enable Λ to be set lower than M_{P} . Since such a strong nonminimal coupling of a scalar field to the curvature is not known, and the fundamental matter fields are fermions, we do not explore the case $\xi_{1s} \ll -1$ further here. Thus, for a realistic model of matter, where several fermionic fields are present, the ultraviolet cutoff Λ is comparable to the Planck mass.

We have obtained the one-loop effective actions for a scalar field and for a Dirac field with arbitrary nonminimal couplings ξ_i and α_j . In order to estimate the magnitude of the induced couplings of the torsion terms, we have to set the magnitudes of the constants ξ_i and α_j . For a scalar field (26) we assume that all the couplings satisfy $|\xi_i| \lesssim 1$, and the coupling to the scalar curvature also satisfies $\xi_1 \leq 1$, which ensures that the induced Newton constant (80) is not too negative, so that the positive contribution from fermionic matter can outweigh it. The given range of couplings also includes the special case $\frac{1}{6}R - \sum_{i=1}^5 \xi_i P_i = \tilde{R}$, where all couplings $10^{-1} \lesssim |\xi_i| \lesssim 1$. In the leading Λ^2 order, that special case is Einstein–Cartain–Sciama–Kibble gravity. Since Einstein–Cartain–Sciama–Kibble gravity resides in the upper end of the chosen coupling range $|\xi_i| \lesssim 1$, it provides an estimate for the effect of the torsion terms in this range. The spin-spin contact interaction in Einstein–Cartain–Sciama–Kibble gravity is weak and becomes comparable to the effect of mass at very high mass densities [69], around 10^{47} g/cm^3 for electrons and 10^{54} g/cm^3 for neutrons. These densities are so high that they are only encountered in black holes and in the early universe, but

¹⁰In this extension of the theory [88], the gravitational action with the vacuum energy sequestering mechanism is considered to be a gauge fixing action S_{gf} for an underlying theory. The BRST-invariant action is constructed as usual by adding the appropriate ghost action S_{gh} . The resulting gravitational action $S_g = S_{\text{gf}} + S_{\text{gh}}$ is found to be not only BRST invariant but also BRST exact, in the sense that $S_g \propto \delta_B F$, where $\delta_B F$ is the BRST transformation of a certain functional F .

they are still much lower than the Planck density at which the quantum gravity effects are expected to dominate. Deviation from the special case of Einstein–Cartain–Sciama–Kibble theory does enable propagation of torsion, which is clearly possible in the generic induced action, but the magnitude of the couplings remains weak. Next we shall give a similar estimation for a Dirac field. We assume the couplings for a Dirac field (57) satisfy $|\alpha_j| \lesssim 1$. This range includes the minimally-coupled Dirac field, which corresponds to the couplings $\alpha_1 = 0$ and $\alpha_2 = -\frac{1}{8}$. Thus, in the given range of couplings, the leading-order torsion contributions in the induced action (70) are of a similar magnitude as in the case of the scalar field analyzed above.

In the low-energy realm of classical gravity, the curvature and torsion terms in the contributions (48) and (73), which are multiplied by $\ln(\Lambda/\epsilon)$ in the effective Lagrangian, are heavily suppressed compared to the leading Λ^2 contribution discussed above. The infrared cutoff ϵ can be chosen to be at most of the order of the mass of the lightest matter particles, which are the neutrinos, so that we can set $\epsilon \lesssim 10^{-3}$ eV. Note that setting ϵ ten or twenty orders of magnitude lower than 10^{-3} eV would still result in $\ln(\Lambda/\epsilon)$ being of the same order of magnitude, so that the present discussion does not depend much on the chosen infrared cutoff. Hence, in the coupling ranges chosen above, the dimensionless net couplings of the higher-order terms ($Riemann^2$, $\square R$, $Torsion^4$, $\square Torsion^2$, $(\nabla Torsion)^2$ and $Riemann \times Torsion^2$) in the effective Lagrangian are of the order one or below, and hence their effect on low-energy physics is marginal (apart from their possible impact on the propagation of torsion). This point of view can be justified by treating gravity as a low-energy effective field theory.

5 Induced gauge theory of gravity

It is known that the two formulations of classical gravity on Riemann–Cartan spacetime are not generally equivalent. In the second-order formulation, the independent variables are the metric and the torsion. Induced gravity in the second-order formulation was considered in Sec. 4, and it was noted that the induced action can as well be written as a functional of the first-order variables. In the gauge theory formulation, which is a first-order formulation, the independent variables are the gauge fields required to achieve a desired local gauge symmetry. Here we consider Poincaré gauge symmetry, so that the gauge fields consists of the vierbein and the Lorentz connection. The two formulations of gravity with both curvature and torsion are equivalent classically for the degenerate case of Einstein–Cartain–Sciama–Kibble gravity (see e.g. [69]), but not for PG generally. The decomposition of a Lorentz connection to a Levi-Civita connection and contortion parts can always be inserted into the field equations of PG. Inserting the decomposition of the connection into the action, however, changes the theory significantly [75].

Next we shall consider the induced gravitational action for PG. Recall that gravitational dynamics does not yet exist at this stage, since the gravitational fields play the role of (classical) background fields, while matter fields are quantized. Only after the one-loop effective action for matter has been obtained, we will choose the independent variables for gravity, which will be the vierbein and the Lorentz connection, after which the variational principle can be applied to derive the gravitational field equations. Since the elementary object of special relativity in the setting leading to gravity on a Riemann–Cartan spacetime is a Dirac spinor [63] rather than a mass point or a scalar field, we primarily consider quantization of Dirac fields in this section.

With the intention of obtaining an induced Poincaré gauge theory of gravity, we start from the Hermitian action for a minimally-coupled Dirac spinor,

$$S_{\text{Dirac}}^{\text{min.}} = \int d^4x (\det e^a{}_\mu) \left[\frac{i}{2} \left(\bar{\psi} \gamma^a \tilde{\nabla}_a \psi - \tilde{\nabla}_a \bar{\psi} \gamma^a \psi \right) - m \bar{\psi} \psi \right], \quad (82)$$

where the Poincaré gauge covariant derivative is defined as $\tilde{\nabla}_a \psi = e_a{}^\mu \tilde{\nabla}_\mu \psi$ with the definition of $\tilde{\nabla}_\mu \psi$ given in (54).¹¹ The action for a Dirac spinor has a similar form as in the second-order formulation of Sec. 4.2 with (57) or without (56) nonminimal couplings, except that now the geometry of the background is determined by the independent variables $e^a{}_\mu$ and $\tilde{\omega}_\mu{}^{ab}$. The minimally coupled action (82) is rewritten as

$$S_{\text{Dirac}}^{\text{min.}} = \int d^4x (\det e^a{}_\mu) \bar{\psi} \left(i \gamma^a \tilde{\nabla}_a - \frac{i}{2} \gamma^a \mathcal{V}_a - m \right) \psi, \quad (83)$$

where the vector component of torsion is defined as $\mathcal{V}_a = e_a{}^\mu e_b{}^\nu T_{\nu\mu}{}^b$. Non-minimal couplings to torsion could be included in a similar way as in Sec. 4. However, we should note that the principle of gauge invariance does not require such nonminimal terms. Therefore, we will consider the minimally coupled case for simplicity.

The one-loop effective action is defined (as in Sec. 4.2) as

$$S_{\text{eff}}^{(1)} = -\frac{i}{2} \ln \det(l^2 D) - \frac{i}{2} \text{Tr} \ln(l^2 D), \quad (84)$$

where the squared differential operator D for the action (83) is written as

$$D = \left[\gamma^a \left(\tilde{\nabla}_a - \frac{1}{2} \mathcal{V}_a \right) \right]^2 + m^2. \quad (85)$$

¹¹Note that in the volume element, $(\det e^a{}_\mu) = (\det e_a{}^\mu)^{-1}$, if one prefers to use the inverse vierbein that appears in the covariant derivative.

The kernel expansion of the operator D for a spinor on Riemann-Cartain spacetime has been studied before, particularly in [90, 91]. Such calculations are based on the decomposition of the connection into torsion-free and torsion components in one way or another. We will adopt a similar approach but with one crucial difference: the relation of the two connections (13) shall be used both ways. First the decomposition of the Lorentz connection $\tilde{\omega}^{ab}$ is used for the derivation of the one-loop effective action. Next the Lorentz connection $\tilde{\omega}^{ab}$ is composed back together, so that the effective action is expressed in terms of the background fields e^a_μ and $\tilde{\omega}_\mu^{ab}$, namely, in terms of the gauge fields of PG. After that we can elevate the said background fields into independent gravitational variables, and thereafter determine their dynamics by setting up the variational principle and deriving the field equations. Varying the action before the full Lorentz connection is composed would lead to inequivalent field equations [75], which would ruin our chances to obtain parity with PG. We avoid this problem with the approach described above.

The operator (85) is written with (13) as

$$D = \left[\gamma^\mu \left(\tilde{\nabla}_\mu - \frac{i}{8} \gamma_5 \mathcal{A}_\mu \right) \right]^2 + m^2. \quad (86)$$

The one-loop effective action is derived in the same way as in Sec. 4.2. Then we write it in terms of the variables e^a_μ and $\tilde{\omega}_\mu^{ab}$. We obtain it as

$$S_{\text{eff}}^{(1)} = -\frac{\Lambda^4}{64\pi^2} \int d^4x (\det e^a_\mu) \text{tr} A_0 - \frac{\Lambda^2}{32\pi^2} \int d^4x (\det e^a_\mu) \text{tr} A_1 \\ - \frac{\ln(\Lambda/\epsilon)}{16\pi^2} \int d^4x (\det e^a_\mu) \text{tr} A_2 + \text{ultraviolet-finite terms}. \quad (87)$$

The induced gravitational action at low energies is defined by the first two terms in the effective action (87), which are obtained as

$$\text{tr} A_0 = 4, \quad (88)$$

$$\text{tr} A_1 = -\frac{1}{3} \tilde{R} - \frac{2}{3} \tilde{\nabla}_a \mathcal{V}^a - \frac{4}{9} \mathcal{V}_a \mathcal{V}^a + \frac{1}{9} \mathcal{A}_a \mathcal{A}^a - \frac{1}{6} \mathcal{T}_{abc} \mathcal{T}^{abc} - 4m^2. \quad (89)$$

Thus, at low energies, we obtain the induced gravitational action as

$$S_{\text{induced PG}}^{\text{low-energy}} = \frac{1}{2\kappa_{\text{ind}}} \int d^4x (\det e^a_\mu) \left(\tilde{R} + \frac{4}{3} \mathcal{V}_a \mathcal{V}^a - \frac{1}{3} \mathcal{A}_a \mathcal{A}^a + \frac{1}{2} \mathcal{T}_{abc} \mathcal{T}^{abc} - 2\lambda_{\text{ind}} \right). \quad (90)$$

This is the low-energy part of the PG action [58] with the relative coupling constants of the terms set to certain values. Thus, we have shown that the

low-energy regime of PG is induced by quantized Dirac fields in Riemann–Cartan spacetime.¹² Note that we have dropped the total derivative term $\tilde{\nabla}_a \mathcal{V}^a$ from the action (90). The induced gravitational couplings in the action (90) are given as

$$\frac{\lambda_{\text{ind}}}{\kappa_{\text{ind}}} = \frac{1}{8\pi^2} \left(\frac{N_d \Lambda^4}{2} - \sum_d \Lambda^2 m_d^2 \right), \quad (91)$$

$$\frac{1}{\kappa_{\text{ind}}} = \frac{N_d}{48\pi^2} \Lambda^2, \quad (92)$$

where N_d is the number of free Dirac fields, m_d is the mass of each Dirac field, and we have omitted the terms of order $\ln(\Lambda/\epsilon)$. What was said about the induced couplings in Sec. 4.3 still apply, including the assessment of the orders of magnitudes for the cutoffs.

The third term in the one-loop effective action (87) provides the high-energy or strong-gravity regime of the induced gravitational action. The dimensionless coupling constants of those higher-order terms are of the order of one or lower, and hence their effect is weak at energies well below the ultraviolet cutoff $\Lambda \sim M_{\text{P}}$. The high-energy part of the induced gravitational action (87) does not exactly match the corresponding regime of PG. In PG, the Lagrangian is defined to be quadratic in the field strengths, namely, in curvature and torsion. Therefore, the high-energy part of the PG action consists of squared curvature terms. On the other hand, the induced action typically contains all terms which are gauge invariant and dimensionally permitted. In the present case, it means that the high-energy part of the induced action can also include quartic torsion terms, as well as terms which involve covariant derivatives, for example, $\tilde{\square} \tilde{R}$. This is in general the case in the effective field theory approach to gravity. One should also note that further contributions to the high-energy part are induced at higher loops. Thus, it is practically impossible to achieve perfect parity with PG at high energies in the induced gravity approach.

¹²For the nonminimally coupled Dirac action,

$$S_{\text{Dirac}}^{\text{non-min.}} = \int d^4x (\det e^a{}_\mu) \bar{\psi} (i\gamma^a \tilde{\mathcal{D}}_a - m) \psi, \\ \tilde{\mathcal{D}}_a = \tilde{\nabla}_a + i\beta_1 \mathcal{V}_a + i\beta_2 \gamma_5 \mathcal{A}_a,$$

where the coupling constants β_i are related to the couplings α_i of the action (57) as

$$\beta_1 = \alpha_1 + \frac{i}{2}, \quad \beta_2 = \alpha_2 + \frac{1}{8},$$

the induced low-energy action is the same as in Eq. (90) except that the contribution $-\frac{1}{3} \mathcal{A}_a \mathcal{A}^a$ of the axial component of torsion is replaced by

$$-\frac{1}{3} (1 - 2\beta_2 + 8\beta_2^2) \mathcal{A}_a \mathcal{A}^a.$$

6 Conclusions and outlook

We have obtained the induced gravitational action on an Einstein–Cartan spacetime by identifying it as the cutoff-regularized one-loop effective action of quantized matter fields. This is the generalization of Sakharov’s induced gravity [1] to a spacetime with both curvature and torsion. When the ultraviolet cutoff Λ is chosen to be comparable to the Planck mass, $\Lambda \sim M_P$, the induced Newton constant (80) has the observed magnitude. As usual, the induced cosmological constant (78) is much too large compared to the observed value, since the vacuum energy contribution is comparable to the square of the ultraviolet cutoff, $\lambda_{\text{ind}} \sim \Lambda^2 \sim M_P^2$. We speculated that it might be possible to use the local vacuum energy sequestering mechanism [87] in induced gravity for setting the correct value for λ_{ind} and avoiding its radiative instability. In a reasonable range of nonminimal couplings for the free matter fields, the contribution of torsion was found to be comparable to that in Einstein–Cartan–Sciama–Kibble gravity. Hence, the effect of torsion is quite weak except in very high matter densities. In general, however, the induced gravitational action is more general than the Einstein–Cartan–Sciama–Kibble theory, which implies that propagation of torsion is possible. In the part of the induced action that dominates at high energies, the dimensionless coupling constants were found to be of the order one, which implies that their effect on low-energy physics is marginal.

Then we have set out to show that the Poincaré gauge theory of gravity (PG) can be obtained by using the Sakharov induced gravity mechanism. We have shown that the quantization of free Dirac fields induces the low-energy part of the PG action (90) with certain relative couplings between the curvature and torsion terms. We conjecture that the result can be generalized to any gauge theory of gravity, in particular to the more general metric-affine gauge theories of gravity. The high-energy part of the induced action was observed to differ from the Poincaré gauge theory of gravity, since it does not contain only squared curvature terms but also terms that involve covariant derivatives and further contributions from torsion. This is to be expected in an approach based on effective field theory, since any term that is both invariant under the given symmetry and dimensionally allowed can, and often will, appear in the effective action. In conclusion, based on our derivation of the Poincaré gauge theory of gravity and the general structure of the effective action, we conjecture that the Sakharov mechanism can be used to induce the action for any gauge theory of gravity.

If we regard that gravity is induced via quantization of matter, what should we do with gravity itself? This is something that Sakharov’s approach cannot address. Most of us believe that gravity should be quantized in one way or another. On the other hand, it is conceivable that space, time and gravity could emerge at a length scale above the Planck length, so that quantization in the conventional sense would be unnecessary. The

fundamental theory behind all that, which would not involve a gravitational interaction, might of course be a quantum theory of some kind. Perhaps the strongest hint towards an emergent nature of gravity is the well known and deep connection of gravity and thermodynamics.

The merit of Sakharov’s idea in this perspective is that, having a theory that includes a curved spacetime and also the quantized matter and gauge fields at the sub-Planckian energies, the gravitational interaction is necessarily produced as well. In other words, gravity emerges as an unavoidable companion of quantum matter.

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